

$$\begin{aligned}
p(x|X) &= \int p(x|\theta)p(\theta|X)d\theta \\
&\propto \int p(x|\theta)p(X|\theta)p(X_{\text{fake}}|\theta)d\theta \\
&= \int p(\{x\} \cup X \cup X_{\text{fake}}|\theta)d\theta
\end{aligned}$$